## ПAmIBIA UПIVERSITY OF SCIEחCE AחD TECHחOLOGY

## FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF NATURAL AND APPLIED SCIENCES

| QUALIFICATION : BACHELOR OF SCIENCE |  |
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| QUALIFICATION CODE: 07BOSC | LEVEL: 7 |
| COURSE CODE: MMP701S | COURSE NAME: MATHEMATICAL METHODS <br> IN PHYSICS |
| SESSION: JULY 2022 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER(S) | Prof Dipti R Sahu |
| MODERATOR: | Prof. S. C. Ray |
|  | INSTRUCTIONS |
|  | 1. Answer ALL the questions. <br> 2. Write clearly and neatly. <br> 3. Number the answers clearly. |

PERMISSIBLE MATERIALS

Non-programmable Calculators

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

## Question 1

1.1 Newton's law of cooling states that the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings
1.1.1 Formulate the differential equation and determine the temperature of the body at any time, t .
1.1.2 A body at a temperature of $80^{\circ} \mathrm{C}$ cools to $60^{\circ} \mathrm{C}$ in 30 min in a room temperature environment of 30 oC . Find the temperature of the body after 16 min .
1.2 Solve the equation

$$
\begin{equation*}
x \frac{d y}{d x}+y(x+1)=9 x ; y(1)=15 \tag{5}
\end{equation*}
$$

1.3 Solve the initial value problem $\mathrm{ty}^{\prime}+3 \mathrm{y}=0$, $\mathrm{y}(1)=2$, assuming $\mathrm{t}>0$

## Question 2

2.1 A series circuit consists of a resistor with $R=40 \Omega$, an inductor with $L=1 \mathrm{H}$, a capacitor with $\mathrm{C}=16 \times 10^{-4} \mathrm{~F}$ are connected with $\mathrm{E}(\mathrm{t})=100 \cos 10 \mathrm{t}$. The circuit initial charge and current are both zero.
2.1.1 Find the charge and current at time $(t)$ in the circuit using the differential equation of the above circuit
2.1.2 Write down the steady state solution of the equation.
2.2 Solve $y^{\prime \prime}+4 y=e^{3 x}$

Question 3
3.1

If $A=\left[\begin{array}{lll}3 & 1 & 2 \\ 1 & 0 & 1\end{array}\right]$ and $B=\left[\begin{array}{ll}1 & -1 \\ 2 & 1 \\ 3 & 1\end{array}\right]$, find $A B$
3.2 Solve the system of equations using Gauss-Jordan elimination method
$2 x-3 y=-21$
$3 x-2 y=1$
$8 x-5 y=-49$
3.3 Find the eigenvalues and eigenvectors of the $3 \times 3$ matrix

$$
A=\left[\begin{array}{rrr}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{array}\right]
$$

4.1 Find the first three Laguerre polynomials from the Rodrigues formula

$$
L_{n}(x)=\frac{1}{n!} e^{x} \frac{d^{n}}{d x^{n}}\left(x^{n} e^{-x}\right)
$$

4.2 Determine the inner product of the following functions in $[0,1]$
(a) $f(x)=8 x$,
(b) $g(x)=x^{2}-1$.
(c) Also find $\|f\|$ and $\|g\|$.
4.3 Given the independent set of vectors: $\mathrm{V}_{1}=\left(\begin{array}{l}1 \\ 1 \\ 1 \\ 1\end{array}\right) ; \quad \mathrm{V}_{2}=\left(\begin{array}{l}0 \\ 1 \\ 1 \\ 1\end{array}\right) ; \quad \mathrm{V}_{3}=\left(\begin{array}{l}0 \\ 0 \\ 1 \\ 1\end{array}\right)$ and
the corresponding orthonormal set

$$
e_{1}=\frac{1}{2}\left(\begin{array}{l}
1 \\
1 \\
1 \\
1
\end{array}\right) ; \quad e_{2}=\frac{1}{2 \sqrt{3}}\left(\begin{array}{c}
-3 \\
1 \\
1 \\
1
\end{array}\right) ; \quad \mathrm{e}_{3}=\frac{\sqrt{3}}{3 \sqrt{2}}\left(\begin{array}{c}
0 \\
-2 \\
1 \\
1
\end{array}\right)
$$

express the vector

$$
B=\left(\begin{array}{c}
3 \\
3 \\
1 \\
-5
\end{array}\right) \text { as a superposition of (i) } V \text { (ii) and } e
$$

